Lesson 11 (3.3)

Today: * Review of sketching $f(x)$ from $f'(x)$ and vice versa
* Rules of Differentiation
* Derivative of exponential function.

Office Hours: MWF 2:45 PM - 4:15 PM

Announcements: * Exam 1 scores posted (Averge = 70%)

* Final Exam: Monday, Dec 15th 1PM - 3PM

* HW 10, 11: Due on Tuesday

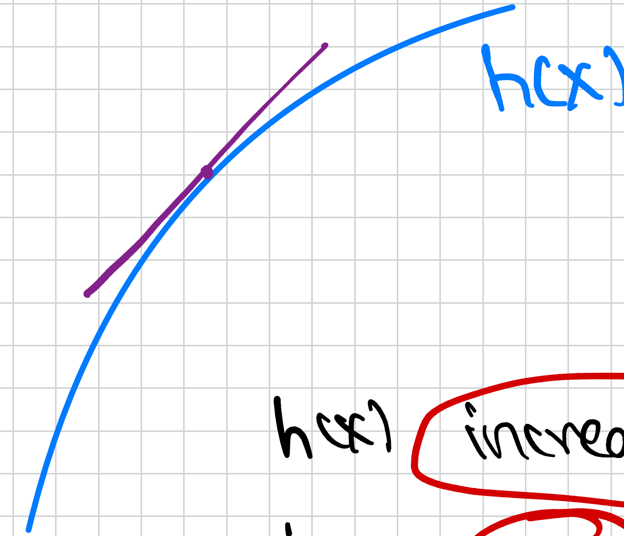
* Quiz 6 (Lesson 9): On Tuesday.

$f(x)$ increasing or Decreasing

$f'(x)$ +ve or -ve

$f''(x)$ increasing or Decreasing

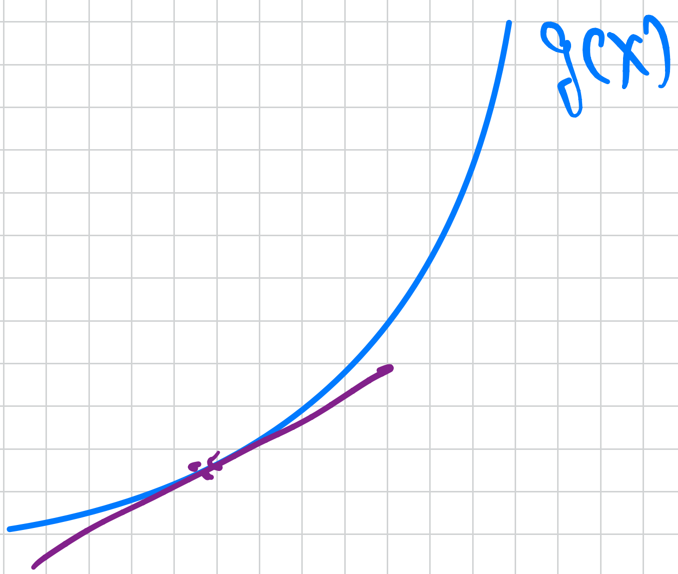
getting closer to zero



$h(x)$ increasing or Decreasing

$h'(x)$ +ve or -ve

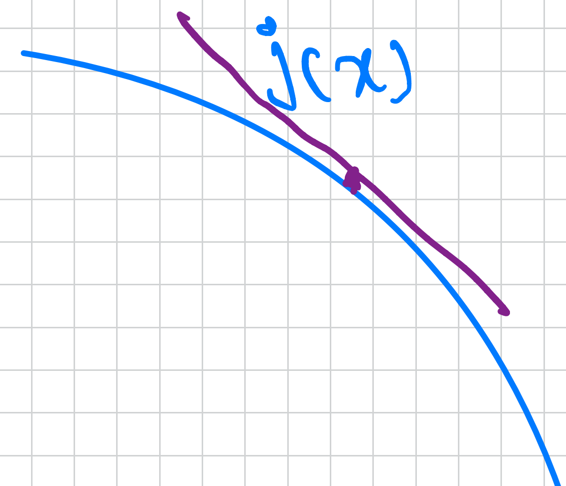
$h''(x)$ increasing or Decreasing



$g(x)$ increasing or Decreasing

$g'(x)$ +ve or -ve

$g''(x)$ increasing or Decreasing



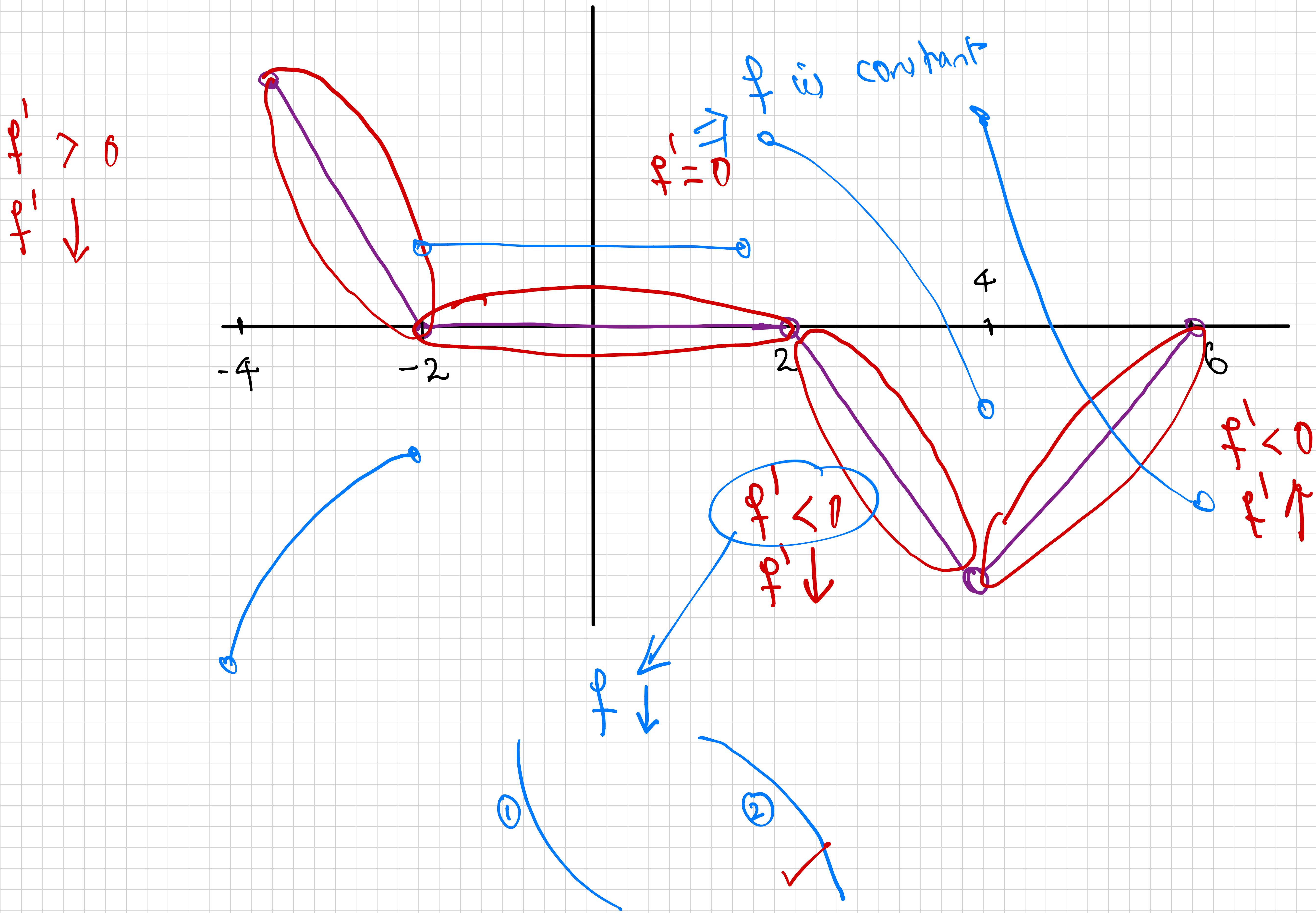
$j(x)$ increasing or Decreasing

$j'(x)$ +ve or -ve

$j''(x)$ increasing or Decreasing

Getting Larger in -ve Direction

Use graph of $f'(x)$ to sketch $f(x)$



Recall!:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Derivative of $f(x)$

Other Notations: $y = f(x)$

$$y' , \quad \frac{dy}{dx} , \quad \frac{d}{dx}(f(x))$$

Rules of Differentiation

① $f(x) = 8$ $\rightsquigarrow$ $\dfrac{d}{dx}(8) = \lim\limits_{h\to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h\to 0} \dfrac{8-8}{h} = 0$

$$\boxed{\dfrac{d}{dx} c = 0, \text{ for any Real number } c}$$

② $f(x) = x$ $\rightsquigarrow$ $\dfrac{d}{dx}(x) = \lim\limits_{h\to 0} \dfrac{x+h-x}{h} = \lim\limits_{h\to 0} \dfrac{h}{h} = 1$

$$\boxed{\dfrac{d}{dx}(x) = 1}$$

③ * $f(x) = x^2 \rightsquigarrow \frac{d}{dx}(x^2) = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$

$$= \lim_{h\to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h} = \lim_{h\to 0} \frac{2x\cancel{h} + h^{\cancel{2}}}{\cancel{h}}$$

$$\boxed{\frac{d}{dx} x^2 = 2x}$$

$$= 2x$$

* $f(x) = x^3 \rightsquigarrow \frac{d}{dx}(x^3) = \lim_{h\to 0} \frac{(x+h)^3 - x^3}{h}$

$$= \lim_{h\to 0} \frac{(\cancel{x^3} + 3x^2\cancel{h} + 3xh^{\cancel{2}} + h^{\cancel{3}\,2}) - \cancel{(x^3)}}{\cancel{h}}$$

$$= \lim_{h\to 0} 3x^2 + 3xh + h^2 = 3x^2$$

* $f(x) = \sqrt{x} \rightsquigarrow \frac{d}{dx}(\sqrt{x}) = \lim_{h\to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$

$$= \lim_{h\to 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} x^{2} = 2x^{1} = 2x^{2-1}$$

$$\frac{d}{dx} x^{3} = 3x^{2} = 3x^{3-1}$$

Recall! $\sqrt{x} = x^{1/2}$, $\frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$$\frac{d}{dx}(x^{1/2}) = \frac{1}{2} x^{-1/2} = \frac{1}{2} x^{1/2 - 1}$$

Generalizing!

$$\boxed{\frac{d}{dx} x^{n} = n \cdot x^{n-1}, \text{ for any } n}$$

④ $f(x) = 7x$ ⇝ $\frac{d}{dx}[7x] = \lim_{h\to 0} \frac{7(x+h) - 7x}{h}$

$$= 7 \cdot \underbrace{\left[\lim_{h\to 0} \frac{(x+h) - x}{h}\right]}_{\frac{d}{dx} x}$$

$$= 7 \cdot \frac{d}{dx} x = 7.$$

$$\boxed{\frac{d}{dx}\left[c\, f(x)\right] = c \cdot \frac{d}{dx}[f(x)]}$$

eg! $\frac{d}{dx}\left[8x^3\right] = 8\left[\frac{d}{dx}x^3\right] = 8 \times 3 \times x^{3-1}$

$$= 24x^2$$

⑤ $f(x) = 5x^4 + 3x^{1/2}$

$$\frac{d}{dx}\left[5x^4 + 3x^{1/2}\right] = \lim_{h\to 0} \frac{5(x+h)^4 + 3(x+h)^{1/2} - 5x^4 - 3x^{1/2}}{h}$$

$$= \lim_{h\to 0} \frac{5(x+h)^4 - 5(x^4)}{h} + \lim_{h\to 0} \frac{3(x+h)^{1/2} - 3x^{1/2}}{h}$$

$$\frac{d}{dx}\left(5x^4\right) = 5 \cdot \frac{d}{dx} x^4 = 5 \times 4 \cdot x^3 = 20x^3$$

$$\frac{d}{dx}\left(3x^{1/2}\right) = 3 \cdot \frac{d}{dx} x^{1/2} = 3 \times \frac{1}{2} x^{-1/2}$$

$$\frac{d}{dx}\left[5x^4 + 3x^{1/2}\right] = 20x^3 + \frac{3}{2} x^{-1/2}$$

$$\boxed{\begin{aligned} \frac{d}{dx}\left[f(x) + g(x)\right] &= \frac{d}{dx} f(x) + \frac{d}{dx} g(x) \\ \frac{d}{dx}\left[f(x) - g(x)\right] &= \frac{d}{dx} f(x) - \frac{d}{dx} g(x) \end{aligned}}$$

eg: $f(x) = \frac{1}{\sqrt{x}} + 8x^7 - \frac{2}{x^3} + 95$, find $f'(x)$.

$$f'(x) = \frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) + \frac{d}{dx}(8x^7) - \frac{d}{dx}\left(\frac{2}{x^3}\right) + \frac{d}{dx}(95)$$

$$\frac{d}{dx}\left[x^{-1/2}\right]$$

$$= -\frac{1}{2}x^{-1/2-1} = -\frac{1}{2}x^{-3/2}$$

$$8\frac{d}{dx}x^7$$

$$8 * 7 * x^6$$

$$2 \cdot \frac{d}{dx}x^{-3}$$

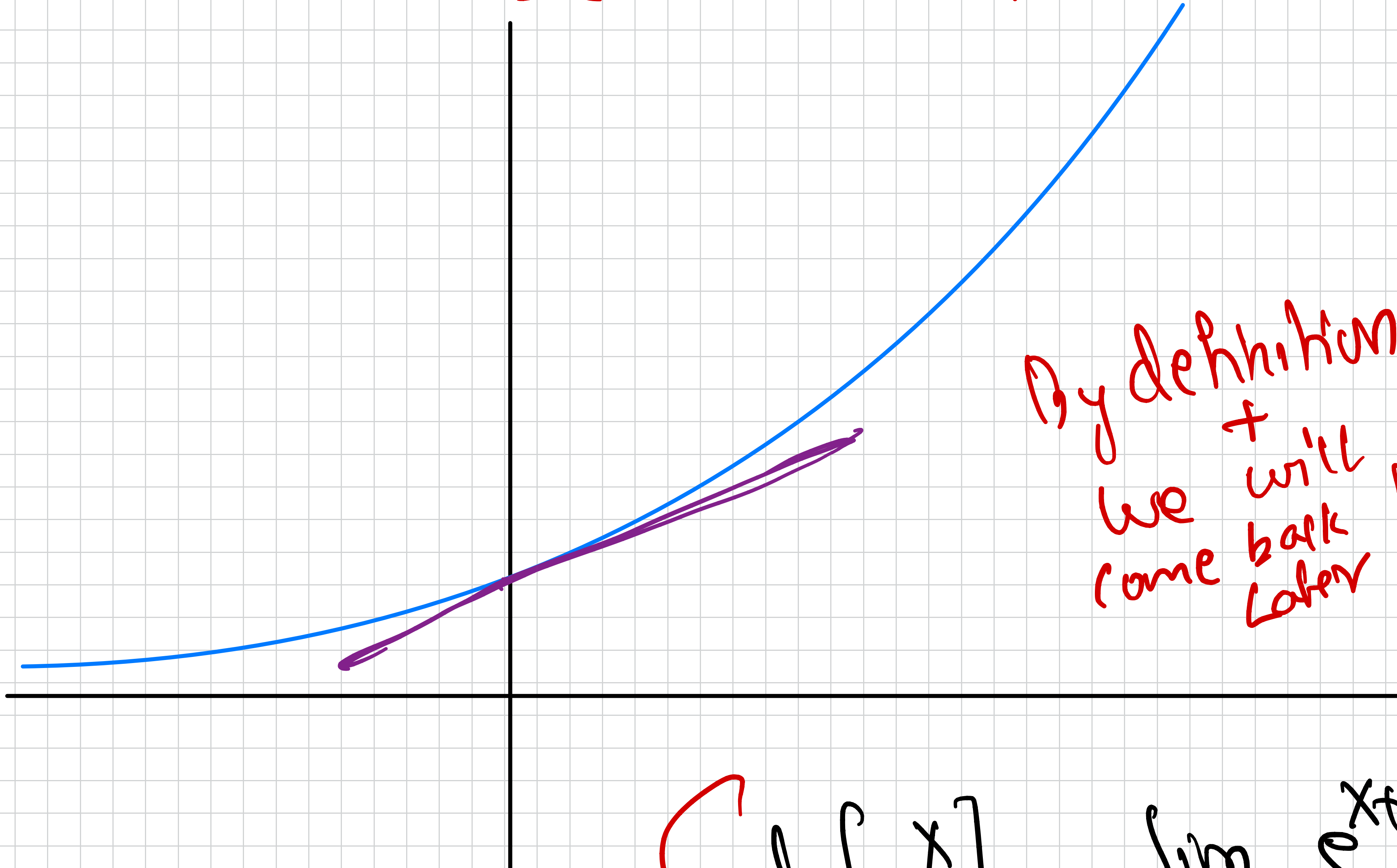
$$2 * -3x^{-4}$$

$$0$$

$$f'(x) = -\frac{1}{2}x^{-3/2} + 56x^6 + 6x^{-4} + 0.$$

$$= \frac{-1}{2x^{3/2}}$$

Derivative of e^x



$f(x) = e^x$

$f'(0) = \lim_{h\to 0} \frac{e^h - 1}{h}$

$=$ slope of tangent line at $(0,1)$

$= 1$

By definition + we will come back later

$$\boxed{\frac{d}{dx} e^x = e^x}$$

$$\frac{d}{dx}[e^x] = \lim_{h\to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h\to 0} \frac{e^x \cdot e^h - e^x}{h}$$

$$= e^x \cdot \underbrace{\lim_{h\to 0} \frac{e^h - 1}{h}}_{1}$$

$$= e^x$$